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Symmetry Breaking in Vortex Flows on Conical Bodies

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Theoretical and experimental results for the appearance of asymmetric flows on conical bodies varying from circular cones to delta wings are reported. Symmetry breaking in the flow is found to be well predicted by theory for circular cones, and the predicted higher resistance to asymmetry for thinner bodies is also supported, although not to the extent indicated by theory. Data are presented on the development of conical asymmetric, and nonconical flows with free vortices on conical bodies, including the effects of vortex breakdown. Vortex breakdown was found to have a significant effect on the onset of asymmetry on thinner bodies. The tests cover a range of *Re* from 20,000 to 3,000,000, with most results at *Re* = 300,000. No asymmetry in vortex core position is found on thin slender wings. Previously reported asymmetries on delta wings are shown to be due to the thickness of the apex of the models tested.

Nomenclature

U = freestream velocity
x,y,z = body axes
α = angle of attack, deg
Γ = vortex strength
ε = nose semi-apex angle, deg
θ = separation angle, measured from equatorial generator
λ = body thickness ratio, cross section: height/breadth

Introduction

THE presence of vortices in the flow over slender bodies at high angles of attack is of importance to the load generation on modern fighter aircraft during extreme maneuvres, to missile aerodynamics, and in the design of the next generations of supersonic or hypersonic transport aircraft. Many tests have established that the appearance of separated vortex flows on slender bodies at high angles of attack can lead to high lateral forces, even at zero yaw. The combination of large forces with a high moment arm means that these effects can be of considerable significance in vehicle directional control. This can cause phenomena such as "phantom yaw," "nose slice," buffet, or other unforeseen vehicle response.

The pattern of flow development on such bodies has been observed in many tests. References 1–12 are a selection of papers presenting results and analyses of such tests. The conclusions may be summarized as follows:

1) At low enough angle of attack, the flow is attached, but at modest angles of incidence, symmetric vortex flow separations evolve on the leeward side of the body.

2) At a larger angle of attack, the vortex pattern becomes asymmetrical. This is correlated with the appearance of side force on the body. It has been established empirically that the value of angle of attack at which side forces first develop is numerically equal to about twice the semi-apex angle of the nose.

3) At higher angles of attack, the asymmetries strengthen, and then evolve into a steady pattern of multiple vortices leaving the body at alternate sides with increasing distance downstream. This process is associated with side forces that first increase rapidly, then undergo a modest overall decline.

4) At still higher angles of attack (>60 deg), the flow becomes unsteady, and at sufficiently high angles resembles the classical Karman vortex stress.

The occurrence of nonsymmetric flows with symmetric onset flow was first found to be surprising. Initial studies proposed that the effect was principally due to nonuniform viscous boundary-layer development, perhaps triggered by small nonuniformities, but this is difficult to justify. Many studies, for example, Lamont and Hunt[2] and Dexter,[9] have demonstrated that the flow structure and direction of side force are very sensitive to small disturbances near the apex. These can be locally large on an inviscid scale. Keener and Chapman[3] suggested that the vortex flow asymmetry was the result of a hydrodynamic instability in the vortex formation process, and argued that there were strong similarities between slender bodies and slender delta wings.

Dyer et al.[13] demonstrated that the calculated levels of side force due to asymmetric separation conditions alone were far too low to provide a satisfactory explanation of the observed results. They also showed how asymmetric solutions for the separated vortex flow over a conical slender body could be predicted from standard line vortex models, even for symmetric boundary conditions. They found that at low angles of attack, the flows were symmetric, but at a critical angle of attack, the solutions exhibited a bifurcation to a nonsymmetry. Symmetry breaking is now familiar in a class of nonlinear physical phenomena as a route to chaos. The theoretical result is of fundamental significance, because it demonstrates that inviscid aerodynamics is a member of this class, while also providing a crucial part of the explanation for asymmetric effects on real vehicles. The theory also suggests the existence of a critical angle of attack, and, therefore, of design criteria that could be used to eliminate or at least minimize lateral forces.

The original work was followed by more complete work by Fiddes and Smith[14,15] and Fiddes[16] using a fuller vortex sheet model, which confirmed the results of the simpler theory. More recently, Marconi[17] found that asymmetries can develop in the flow computed via solution of the Euler equations for the flow around a cone, thus, again confirming that the appearance of asymmetry is an inviscid phenomenon. Marconi also found that past the bifurcation point where asymmetries

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first appear, symmetric solutions seemed to be unstable, and that if symmetry conditions were relaxed, the asymmetric solution dominated.

Both line-vortex and vortex-sheet flowfield models are based on conical slender body approximations, and, therefore, predict that phenomena will be a function of the ratio of angle of attack to semi-apex angle, giving a critical value of this parameter of just over two for a circular cone. This shows excellent correlation with the empirically determined condition for the appearance of side forces mentioned above. The extended theory by Fiddes and Smith[15] suggested that flatter bodies; for example, elliptic cones, would be more resistant to the appearance of asymmetries. In particular, their theory indicated that at sufficiently low thickness-chord ratios no asymmetry would occur. This conclusion is of special interest, because it is in direct opposition to the suggestion of Keener and Chapman[3] that similar effects occur on slender wings or bodies. Woolard[18] attempted to derive a mathematical relationship between the wing and cone flows, but this analysis contained an error, and the flows cannot be mapped in the manner described. However, several workers have reported asymmetries and/or unsteadiness in the flow on slender delta wings at high angles of attack, which might be presumed to result from mechanisms related to those on slender circular bodies.

The theoretical results are important for a second reason, in that inviscid vortex mechanisms may be expected to be good candidates for useful experimental study at low Reynolds number. This is supported by the experimental work of Keener[10] on ogive cylinders. He comments, "Reynolds number and Mach number do not have much effect on the onset angle for the different vortex regimes." The objective of the present work was to obtain experimental results that could be compared with the predictions of the theory, in particular, to identify the critical angle of attack at which symmetry breaking occurs. A second objective was to clarify the effect of body thickness on the flow.

The great majority of experimental studies of vortex flows on slender bodies have concentrated on tangent-ogive cylinders. These shapes are characteristic of those in practical use. Theoretical studies, however, have concentrated on simpler conical bodies, extrapolating predictions to improve the understanding of the flows around more complex shapes. There has been comparatively little experimental work on purely conical bodies.[4,11,16] Thus, the aim of the experiments reported in this paper is to examine the onset and further development of asymmetric flows on conical bodies as a function of angle of attack, semi-apex angle, and body thickness. The experiments were undertaken on a family of bodies, which were chosen to give a direct comparison with the theory, including circular and elliptical cones, and slender delta wings.

Theory

The original papers of Fiddes and Smith[14,15] did not include an extensive parametric study of their theoretical predictions, particularly for the effect of body thickness. One of the first tasks of the current work was, therefore, to establish the bifurcation solutions over a range of relevant parameters. As has already been mentioned, the original theory[13] represented the flowfield with isolated line vortices, following the work of Bryson.[12] Later work[14,15] used a full vortex-sheet model. This give more accurate predictions of vortex positions and strengths, but gave essentially similar results for the appearance of asymmetries in the flow.

The near singularity of the Jacobian matrix of partial derivatives that is inverted and used in the Newton iteration scheme causes difficulties when obtaining solutions for either model close to the bifurcation from symmetric to asymmetric flow. Comparisons between the two models showed that predictions of bifurcation angle were similar, and the differences were believed to be within the anticipated range of experimental accuracy. In order to simplify the work, the simpler line-vortex model was chosen for the present analysis.

For the line-vortex (Fig. 1), the conical flow past a circular cone is represented by two line-vortices of unknown strength, introduced along conical rays whose positions are also to be determined. A condition analogous to a Kutta condition is applied at the prescribed separation lines to make them streamlines of the three-dimensional flow, and the transverse component of the force acting on each vortex is balanced by an equal and opposite force acting on the "cut" connecting it to the adjacent separation line. The assumptions of slender-body theory are made and the problem reduces to the solution of two-dimensional Laplace's equations in cross-flow planes. The conical flow assumption is a similarity solution, which means that the problem needs be solved in only one cross-flow plane, with the flow in any other plane being found by simple scaling. The problem is independent of Mach number, apart from the calculation of pressures that are relative to a Mach number-dependant datum. Detailed descriptions of the model, governing equations, and the assumptions made are given in Ref. 13.

Bifurcation studies are carried out by varying one parameter of the problem with all other parameters fixed. The parameters of this problem are the incidence ratio (the ratio of angle of attack to nose semi-apex angle), the position of separation lines, and the thickness ratio of the body, defined as one for a circular cone and zero for a flat plate.

Bifurcation studies for a series of shapes were carried out. As found previously, the effect of varying separation positions was modest, so the bifurcation limits of Fig. 2 could be defined. The bifurcation limit was found to be in the area shown over the range of separation angles studied. The separation angles indicated are defined leeward of the equatorial generator of the cone, as shown in Fig. 1. Figure 2 confirms that the incidence above which asymmetric solutions exist strongly depends on body thickness ratio. Varying symmetric separation positions for thick bodies; for example, circular or almost circular cones, can influence the onset of asymmetric flow to some extent. For many thinner shapes, the range of possible separation positions for which solutions exist is small. Many of the separation positions for which solutions exist do not agree well with experiment. However, the corresponding values for vortex positions are much more representative, and because the line-vortex model does not take into account the vorticity in the vortex feeding sheets, the use of realistic separation positions is not important.

The physical implications of the predictions made by the bifurcation curve require careful consideration. The theory assumes conical flow and slender body theory, which allows no upstream influence and also takes no account of phenomena such as vortex breakdown. The large values of incidence

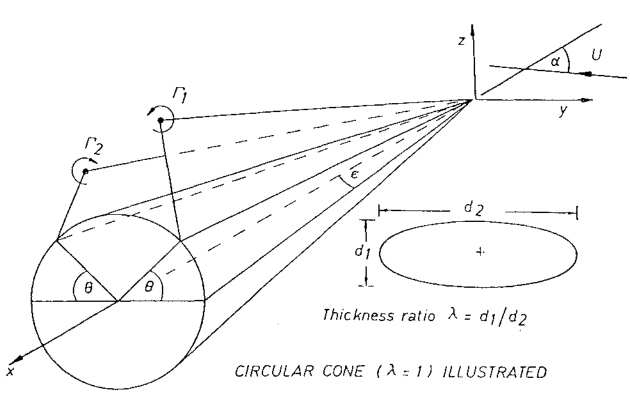


Fig. 1 Theoretical model.

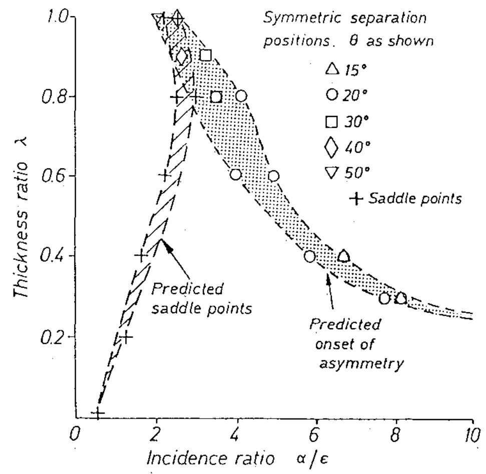


Fig. 2 Bifurcation limits.

ratio infer very high angles of attack for all but the most slender of forebodies and the strongly three-dimensional flows that result are not within the scope of the theory. Despite this, the results are of great interest for configurations with slender forebodies.

Recent papers by Fiddes and Williams[19] and by Williams[20] demonstrate that the basic results apply, in modified form, to a wide variety of slender body shapes. These papers provide a mechanism for reinterpreting the results of the present paper for other fuselage cross sections. Pidd and Smith[21] have reported a detailed study of the existence of symmetric and asymmetric solutions for the vortex flow on circular cones. They demonstrate that the appearance of stable asymmetric solutions concides with the instability of the symmetric solutions, so that bifurcation occurs "with exchange of stabilities." Their detailed results are consistent with the results presented here.

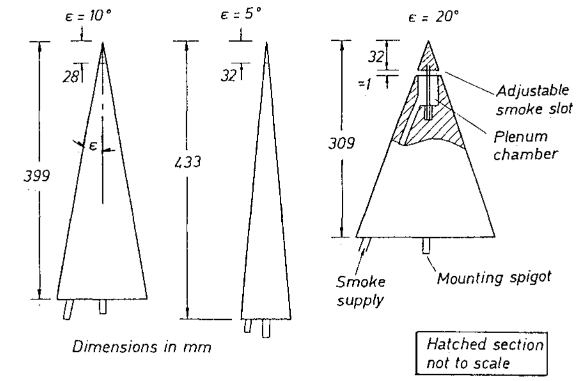
Another point of theoretical interest is the direction of the surface flow near the leeward meridian. At low angle of attack, the flow around the vortex will reattach to the upper surface, but at a sufficiently high angle of attack, the flow will be unable to reattach and form a saddle point in the flow. Peake et al.[5] suggested that an instability associated with the existence of saddle points in the flow was a cause of the appearance of asymmetry.

A study of the appearance of saddle points in the flow was carried out and the results are also presented in Fig. 2. It will be observed that on a circular cone the appearance of a saddle point and asymmetry are indeed quite closely correlated. However, it will also be observed that the saddle point considerably precedes vortex asymmetry for thinner bodies, so there can be no causal link between these two phenomena.

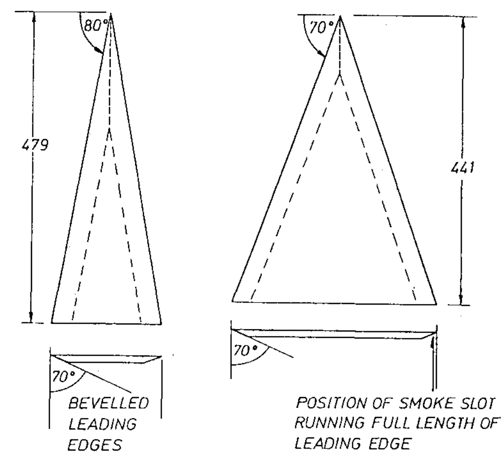
Experimental Setup

A selection of forebody shapes and delta wings were tested in the Aerospace Engineering Departments 0.8 m × 0.6 m closed return tunnel with a working range of 1–100 m/s. The tunnel chosen had been specially designed with a contraction ratio of 12:1 to achieve flows with turbulence levels less than 0.5%. Most tests were carried out at 10 m/s. Flow speeds were measured with a commercial hot wire meter, calibrated against a vane anemometer.

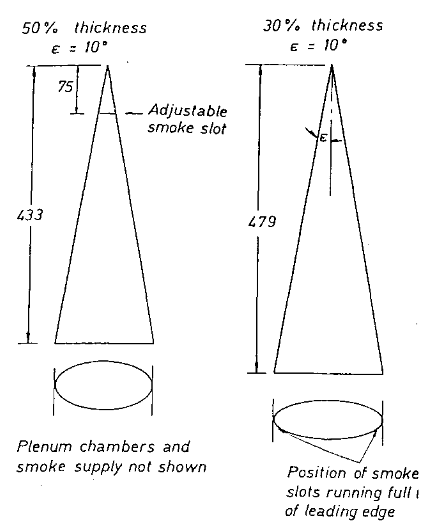
The models tested included three circular cones having 5-, 10-, and 20-deg semi-apex angles, two delta wings of 10- and 20-deg semi-apex angle previously tested by Lowson,[22,23] and two elliptical cones with minor/major axis ratios of 0.5 and 0.3 both with semi-apex angles of 10 deg (Fig. 3). The models were mounted on a parallel arm arrangement so that incidence could be easily adjusted from outside the tunnel. The models were tested at incidences ranging from 0 to 90 deg.



a) Circular cone models



b) Delta wing models



c) Elliptical cone models

Fig. 3 Wind tunnel models used.

The principal flow visualization medium in the current investigation was smoke, and all these models incorporated a plenum chamber and slots to allow smoke emission. The three circular cone models were of similar construction (see Fig. 3a). The models were turned from solid blocks of hardwood and the nose cones machined from aluminium. The front section of each model was hollowed out to create a plenum chamber where smoke could circulate and exit uniformly.

Smoke was emitted via a circular slot around the cone apex that could be adjusted in width, altering nose geometry slightly as a consequence. The two delta wings had sharp leading edges bevelled at 20 deg on the underside. The smoke outlet on these models was a slit 0.2 mm wide, uniformly machined 1 mm below the entire leading edge, and passing into a plenum chamber. The overall thickness of the wings was 12 mm (see Fig. 3b). The two elliptic cone models are illustrated in Fig. 3c. The thicker elliptical cone shared the nose apex-type emission of the circular cone models, whereas the thinner elliptical cone had a slot similar to the wing models.

The flow was illuminated with a variety of sources. An argon-ion laser operating in all lines mode at around 6 W was used to produce a light sheet via a small cylindrical lens. The models were illuminated from the side or from above to avoid any flow features being hidden in the shade of the body. An optical bench setup provided beam steering to achieve a light-sheet of specific height and chordwise position. The light-sheet could also be placed at a prescribed angle of incidence in the tunnel, and for the majority of the work, it was inclined parallel to the cross-sectional plane of the cones/wings. Flash and flood lights were also used, the latter being particularly useful for studying smoke trails and identifying vortex breakdown.

Photography was accomplished with a variety of cameras, but principally by a 35-mm camera. The laser pictures were taken typically at an exposure of around 1/2000 s and at f1.4. Kodak T-Max 3200 ASA film was used and push developed often as high as 50,000 ASA.

Surface flows were studied by means of oil and dye flow techniques and a limited investigation of the side forces present was also possible by attaching strain gauges to the model supports. No force or pitching moment readings were taken because of the wealth of experimental data already in this area.

Experimental Observations

The results obtained in the present experiments proved to be rather complex. As found by other experimenters as well, the results were sensitive to quite small disturbances in experimental conditions. For example the presence of upstream smoke probes was found to produce marked variations unless careful precautions were taken. Small variations at the nose of the model were found to be significant. Particular care had to be taken not to cause disturbance through the introduction of the smoke at the nose. The results were also, not surprisingly, sensitive to transition processes. However, despite these confusing factors, a reasonably clear picture of the fundamental mechanisms emerged.

Study of Circular Cones

Figure 4 shows a sketch of the main sequence of symmetry breaking as a function of angle of attack. The basic process was common to all cases, but the complete description below centers on the 5-deg circular cone.

A) At low enough angles of attack, no separation is visible, but as the angle of attack is increased, symmetric separation appears in the form of shear layers feeding a pair of vortex cores in the form of the classic tightly rolled vortex.

B) The vortex cores appear initially toward the top center of the body and move outward and upward as the angle of attack increases.

C) At some angle of incidence asymmetries start to appear. The exact angle of attack at which this occurs is obviously a matter of experimental judgment, but it is usually fairly clear. As angle of attack is further increased, the asymmetry grows, so that the feeding sheet to the higher vortex gradually extends into the leeward flow. The flow at this stage could be seen to be essentially conical.

D) Eventually one feeding shear layer shows signs of instability and tends to form into a second separate vortex closer to the body. Because the topmost vortex can no longer be fed directly by the separating shear layer, conicality conditions cannot now apply.

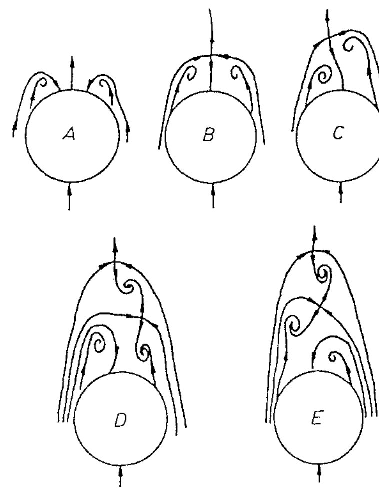


Fig. 4 Main sequence of flow states observed.

E) With a very slight increase in angle of attack, the upper vortex detaches itself from the lower and is "captured" by the vortex on the other side of the body. This capture process, identified by movement of the connecting stream surface, is simultaneous over the entire length of the body visualized by the smoke. There are now three distinct vortices, with the uppermost lying almost parallel with the freestream. If angle of attack is held near to this capture condition, there is a strong tendency for unsteadiness and, thus, buffet, with the vortex forms switching between *D* and *E*.

Further increases in angle of attack were observed to result in additional vortex formation and further alternating capture processes. These high-incidence flowfields are described in greater length in Ref. 24.

The "streamlines" presented in Fig. 4 correspond to sections through stream surfaces in the cross-flow plane. The smoke visualizations of the present tests enable the patterns shown to be developed as representative of the flows observed on the present experiments.

A key feature of these patterns is that the free vortex structures are always observed to be linked to one of the vortices remaining attached to the body. The observation of smoke filaments joining vortices requires that no streamline can pass between them.

Some photographs showing laser light-sheet visualizations of the process for the 5-deg cone (flow states *B*, *C*, *E*) are given in Fig. 5, and critical angles for the vortex formation process are plotted in Fig. 6. It was found that the effect of smoke on the process often could be significant. Thus, the data presented in the report were taken with minimum smoke levels. For the photographs greater levels of smoke were required, with some effect on the angles at which the flows were observed. Observations of the onset of asymmetry were based upon judgements of the flow condition. This data could normally be matched to better than ± 1 deg on repeat runs, though some tests did show larger variations. Accuracy to greater than ± 1 deg is not claimed.

The process described above is believed to be the essential form of the flow development on slender conical bodies at high angles of attack. The same sequence was observed for the 10 deg cone, but at almost double the angle of attack, with the consequence that alternate vortex separation was not observed, even at the highest incidence tested. Instead, a complete breakdown of the flowfield occurred above an angle of attack of about 70 deg. For the 20-deg cone, the separated flow was found to be nonconical at lower angles of attack.

Fig. 5 Photographs of the flow states on a five degree cone.

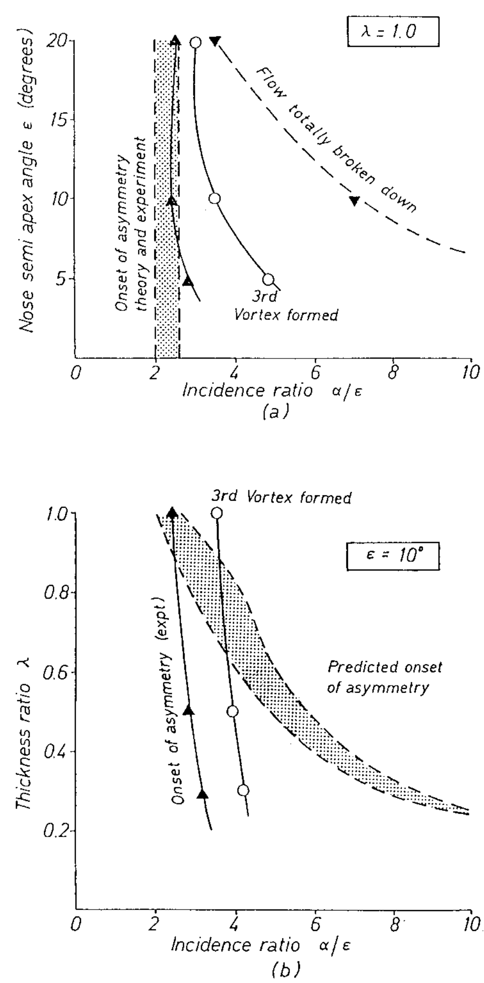


Fig. 6 Vortex formation on conical models.

It will be recalled that saddle points were also studied in the theoretical part of this paper. It was found that the appearance of a saddle point was closely coincident with the appearance of asymmetry in the vortex position for a circular cone. In the visualization experiments it was also found that the appearance of a "stalk" in the smoke leading away from a circular cone was virtually coincident with the first appearance of asymmetries. On the thinner bodies and delta wings, the stalk would occur with symmetric vortex flows, confirming the independence of saddle point and asymmetry suggested by the theoretical results of Fig. 2.

Study of Elliptical Cones

Further studies were undertaken on the 10-deg elliptical cones. The 2:1 ellipse gave results similar in essence to the circular cone, but with some important differences. The basic main sequence process of symmetry/asymmetry/multiple vortex and vortex capture was again observed. However the transition between states was less clearly marked, and buffet was also comparatively minor. The most significant modification to the circular cone was the important role of vortex breakdown phenomena in this case. In tests at all speeds, the first appearance of asymmetry was closely followed by the appearance of vortex breakdown in the lower vortex toward the trailing edge of the body. As the angle of attack was increased vortex breakdown affected all the vortices observed.

Vortex breakdown moved toward the wing apex as angle of attack increased, in the same way as is now familiar on delta wings. The vortex capture process was similar to that observed for the circular cone, but the vortices involved were clearly broken down over much of the wing. At high enough angles of attack (>65 deg) the various areas of the flow behind the wing became rather indistinct so that only a general area of stagnant wake flow could be observed. Figure 7 shows how this wake flow developed from successive breakdown of the vortices present. For all the cones tested, it was the vortex closest to the model surface that burst first. Vortex asymmetry

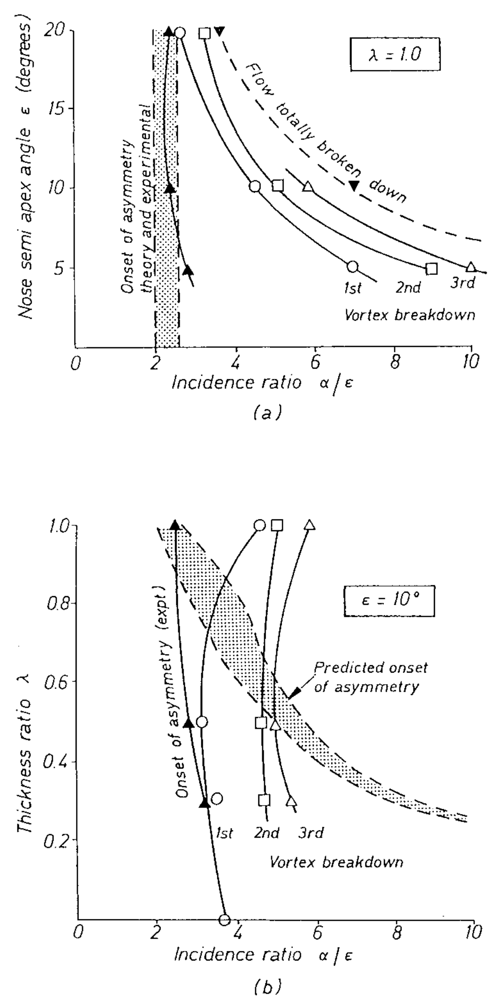


Fig. 7 Vortex breakdown on conical models.

was present for both the 30% and the 50% thickness elliptical cones, although closely linked to vortex breakdown.

The observed flows on the 20-deg circular cone and the elliptical models were similar in many respects. This is believed to be due to the near coincidence of vortex breakdown and the onset of vortex asymmetry for these bodies.

Studies of Delta Wings

Tests were also carried out on 20- and 10-deg semi-apex-angle delta wings. Visualization demonstrated that no overall asymmetry in vortex core position was present in the flow at any speed. In contrast to the elliptical cone results, vortex breakdown toward the trailing edge was not associated with observable asymmetries in the vortex positions further forward. However, as previously noted by Lowson,[25] vortex breakdown did not normally occur at the same chordwise location on the wing, particularly on the more slender wing. Lack of symmetry in vortex breakdown position would result in rolling moments being imposed on the wing. At still higher angles of attack, the asymmetries induced by vortex breakdown tended to disappear, with the flow taking on the appearance of a stagnant bubble separation. This separation was bounded by shear layers that, in turn, supported instabilities of Kelvin-Helmholtz type. The angle of attack at which this occurred was a weak function of speed, but typically was about 55 deg.

These results are consistent with the theoretical predictions, however, several workers have reported either asymmetries or periodic unsteadiness in the flows around delta wings at high angles of attack. Therefore, the original references have been studied. Bird[26] reported an asymmetrical flow on a delta wing of 45-in. chord and 3.5-deg semi-apex angle. This result has been used by several workers to infer similarities between the flows on thick slender bodies and delta wings. In fact, in Bird's work, "the flat-plate airfoil section was obtained by constructing the wings from 3/4 inch (19.05 mm) plywood and rounding the leading edge." Thus, for the highly swept wing tested by Bird, the thickness was greater than the span for the first 12% of the chord. It seems safe to conclude that the asymmetries observed were a result of features associated with apex of the model rather than with the wing as a whole. Results from Fellows and Carter[27] also show very clear vapor screen evidence of asymmetric flows above 17 deg on a delta wing of 4.5-deg semi-apex angle at M 2.8. Again the apex of the wing has finite thickness, including a supporting spine. For angles of attack above 17 deg, this spine will not lie within the Mach cone, so that viscous separations could also fill the underside of the wing. It is also clear from the vapor screen photos that all the asymmetric cases contain cross-flow shocks.

The original paper by Keener and Chapman[3] based the suggestion of similarities between wing and body asymmetries primarily on the evidence of induced rolling moment on wings at zero yaw, as measured by Shanks.[28] The models used by Shanks were supported by a faired spine, which on the smallest semi-apex angle wings (6 deg) had a diameter of about one third of the local span over the front half of the model. The models were constructed from 0.125-in. aluminium with semicircular leading edges. Thus, again it can be seen that the delta wing proper was preceded by a nose that was not of delta form.

Figure 8 gives a diagram of the shapes of the apex at the 10% chord point for each of the cases noted above. The drawings are scaled so that they correspond to wings of equivalent chord; that is, the span is proportional to the semi-apex angle. The shapes have been drawn as far as possible from details given in the original papers. In the case of the wing used by Bird, a plausible extrapolation of the rounded leading edge is shown. In the case of the Fellows and Carter work, the shapes have been determined from the original drawings, kindly retrieved by S. Fellows of the Aircraft Research Organization. It can be seen that in every case the apex shape is far from the ideal delta wing form.

It has been noted already that the onset of asymmetries on a slender body is found to be dominated by the shape of the initial part of the nose. It seems reasonable to expect that a similar effect would occur on a slender wing. Thus, the relationship between circular slender bodies and delta wings suggested by Keener and Chapman[3] and others is believed to represent a relationship between the shapes of the apex in each case, rather than between the body and wing.

In order to test this hypothesis, a special experiment was performed on two slender delta wings of 5-deg semi-apex angle. Wing I was 3.3 mm thick with a 45-deg chamfer, and wing II 1.3 mm thick with a 30-deg chamfer. Chords were 435 and 444 mm respectively. The form of the apex at 10% chord in each case is given in Fig. 8. Wing I was of prismatic section forward of 9% chord, and Wing II forward of 6% chord. Neither wing had an internal smoke supply, which could have been a possible source of anomalies, so the tests were performed with "antismoke" visualizing the clear hole left by the centrifuging of smoke particles from the vortex cores. This proved to be well suited to the problem.

The results obtained are shown in Table 1. This lists the angle of attack at which a third vortex appeared in each case. Also shown is the typical angle of attack at which vortex breakdown first appeared at the trailing edge (this condition is subject to some margin and to effects of hysterisis). The results are shown for two cases; with the wings as described, and with the wings inverted, so that the chamfer was facing in the opposite direction.

Table 1 demonstrates large differences in the angle of attack for which a third vortex appeared; that is, the loss of conical flow. Because all the wings had the same planform, this is strong confirmation of the postulated critical effect of apex shape on the results. At angles of attack somewhat below the

Table 1 Test results for 5-deg delta wings at 10 m/s

	Wing I, deg		Wing II, deg	
	Up	Inverted	Up	Inverted
Third vortex appears	35	31	44	39.5
Vortex breakdown at trailing edge	32	31	36	37

Note: All angles of attack are given to the nearest 0.5 deg.

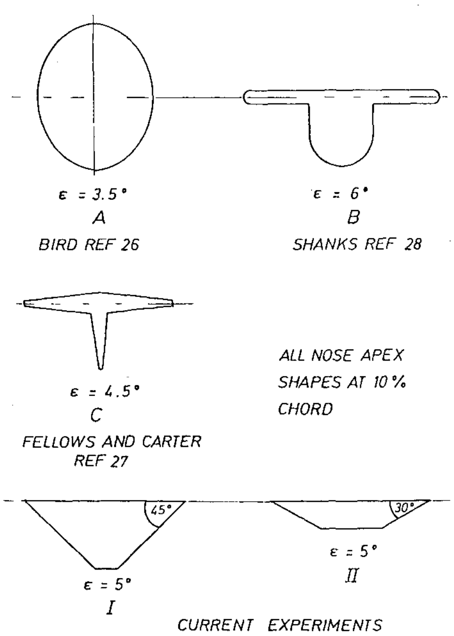


Fig. 8 Comparison of apex shapes for some experiments on slender deltas.

critical angle, the visualization technique showed that the vortices over the prismatic part of the apex were asymmetric. However, this loss of symmetry was rectified as the vortices passed down the wing. This may be contrasted with the opposite effect on circular bodies where initial asymmetries are strongly amplified as they pass down the body.[16]

The results also demonstrate that the plane side up cases (as drawn in Fig. 8) were more stable than the inverted wing. Fiddes and Williams[19] have demonstrated that a triangular body shape is less liable to asymmetries when the flatter side is upward. Because it is believed that results of the present experiments were dominated by apex shape, those theoretical predictions are confirmed. Note also that if the effects scale on the ratio of angle of attack to semi-apex angle, then any effects on the 10-deg delta would occur at unreasonably high angles of attack on any sensible form of apex.

Table 1 also shows that vortex breakdown on these wings is affected by the apex shape. This is not surprising, because the vorticity distribution toward the center of the core will be a direct function of the vorticity shed in the region of the apex. During the experiments, it was also observed that the appearance of a third vortex frequently had strong effects on the vortex breakdown, sometimes inducing it if not present. (In the case of the 30% elliptic cone, the effect of the appearance of the third vortex was actually to remove an existing vortex breakdown.)

Theoretical models for the flow on delta wings are based on idealized, infinitely thin bodies. Unfortunately, experiments can only be performed on real bodies, which must always violate conicality conditions at the apex to some degree. Thus, the perfect experiment to test the theory cannot be performed. Within this limitation, the present experiments seem to confirm that no asymmetries in vortex position are present in the flow over an ideal slender wing. This conclusion is entirely consistent with theory. Furthermore, several features of the theoretical predictions for the appearance of asymmetries on thicker bodies are supported by the present experiments. Thus, it is believed that previously observed asymmetries for delta wings were due in each case to the departure of the apex shapes from ideal delta form. (It is also possible that the observed onset of asymmetry on the elliptic cones is due to inaccuracies in manufacture of the apex region.)

It may also be noted that further examination of the original Shanks[28] data on delta wings suggests that nonzero rolling moments at zero yaw angle, and other rolling moment anomalies, are present for higher apex angles than suggested by Keener and Chapman.[3] The results of this re-analysis are shown in Fig. 9. Also shown in Fig. 9 is the anticipated onset of vortex breakdown on delta wings as suggested by the present tests. The lack of symmetry in vortex breakdown position, already discussed, is a reasonable alternative cause of rolling moment on a delta wing.

Discussion

The original objective of the present experiments was to compare the onset of flow asymmetries with theory. The results from the present experiment are shown together with the theoretical results in Fig. 6.

These results show broad agreement with the theoretical predictions for circular cones. Perhaps the most important result is to verify that asymmetric but still conical flows can exist on conical bodies, as predicted by theory. Note that this conclusion is also verified by the conical surface flow patterns on cones observed by Peake et al.[5] The change from symmetric to asymmetric flow occurs at an incidence parameter of between 2 and 2.5 for each case. This is in reasonable agreement with theory, and is also consistent with empirical observations on slender ogive cylinders.

The present results have been obtained on bodies of purely conical shape. These results will have some application to

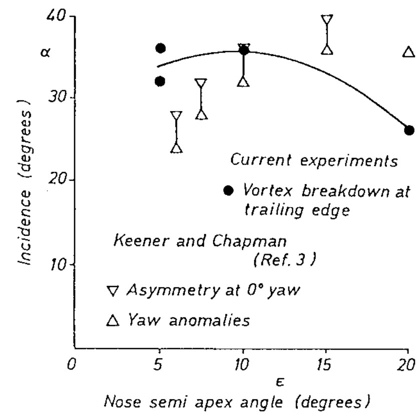


Fig. 9 Re-analysis of data from Shanks.

conical forebodies. The effects of the base on asymmetries and the pattern of vortex development seemed to be small. These processes are believed to be unaffected by the presence of an afterbody. However, vortex breakdown effects would be affected directly by an afterbody, and the results could not be expected to be characteristic of cones with afterbodies.

It has already been noted that Fig. 6 shows good agreement between theory and experiment for the case of the circular cone. For thinner bodies, experiment shows an initial asymmetry at a lower angle of attack than predicted by the theory. However, it will be recalled that for the elliptic cones, the asymmetry was closely associated with the appearance of vortex breakdown in one vortex core, as demonstrated in Fig. 8. Thus, the experimental results may be associated with phenomena not included in the theory.

It will also be noted that nonconical flows with free vortices were observed under conditions where theory suggested symmetry. Although some increased tendency toward symmetry was observed for the thinner bodies, this effect was not as strong as predicted by the theory.

For delta wings, no asymmetry in vortex crossflow position was found, but a variation in the chordwise location of vortex breakdown was observed. It has been noted[25] that vortex breakdown on a delta wing does not occur at the same chordwise location, particularly for the more slender wings. Tobak and Peake[8] used the results of Lowson[25] to argue that this phenomenon was an example of a bifurcation process. It is intriguing that the flows on conical bodies seem to be subject to two separate symmetry breaking processes: one due to the basic asymmetry in vortex core position, applying for thicker bodies; and the other due to vortex breakdown asymmetries that dominates the thinner. It is not surprising that the flows on these bodies have been described as "infuriating."[7]

The experiments seem to demonstrate that flow on an ideal delta wing would remain symmetric. It was also concluded that no major unsteadiness was present, although this conclusion is less certain because no instrumentation was used to examine possible periodic effects. However, the visual observations were very clear. At low speeds it is certain that no gross periodic flow was present at high angles of attack. The wake behind the wing could be seen to be virtually stagnant. The bounding shear layer did support Kelvin-Helmholtz instabilities, which clearly could be seen traveling up the wake edge, in a very similar manner to that reported in Refs. 22 and 23. At higher speeds visual observations are not necessarily reliable, but the clear impression was formed that no major wake disturbance was present. The unsteadiness observed was confined to a region close to the edge of the wake, which still appeared rather stagnant. This conclusion is not consistent with the experiments of Redinotis et al.,[29] who found strong periodic wake flow above a critical angle of attack of around 35 deg on a delta wing of 14-deg semi-apex angle. It can at least be concluded that the effects noted by Redinotis et al. are not a necessary feature of the flowfield.

A. L. Williams has told the authors that her theoretical investigations of conical vortex asymmetries demonstrated a second limitation in the solution as the feeding vortex sheet "developed a point of inflection." This occurred at a ratio of angle of attack to semi-apex angle of 5.2. This description of the mathematical model is virtually identical to the effects observed in the tunnel at stage *D* of the main sequence described in Fig. 4. It will be noted from Fig. 6a that the appearance of the third vortex is at an incidence ratio of 5.0 in the present experiments on a 5-deg cone.

During the final preparation of this paper, a report by Stahl et al.[30] was received. Flow visualizations in a water tunnel on highly slender delta wings are reported. No evidence of asymmetry was found, and it was concluded that previous results had been dominated by the effects of apex shape, which agree with the present findings.

Conclusions

The present work was undertaken to explore the predictions of the theory for symmetry breaking in the flows on conical bodies. The theory suggests the following:

1) Flow with conical asymmetries are possible.

2) The onset of asymmetry on conical slender bodies should depend upon the ratio of angle of incidence to semi-apex angle.

3) There should be a strong effect of thickness on bifurcation point.

4) No asymmetry exists for thin enough wings; for example, slender delta wings.

The experiments indicate the following:

1) Flows with conical asymmetries do occur, and will give significant side force.

2) The onset of asymmetry on conical slender bodies is fairly well predicted by the ratio of angle of incidence to semi-apex angle. There is some effect of semi-apex angle and of speed.

3) Thinner elliptic cones are resistant to asymmetry, but not to the extent predicted by theory. Vortex breakdown effects confuse the situation and are a further possible cause of asymmetries.

4) Slender wings do not support asymmetric flows, except via chordwise position of vortex breakdown. Earlier results showing asymmetries on highly swept delta wings simply reflect the thickness of the apex tested.

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